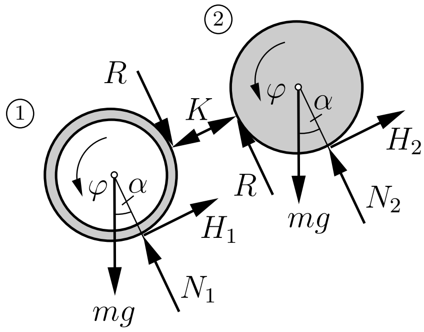
Lösung - Aufgabe 1

a) Freikörperbild:



Massenträgheitsmomente: $\Theta_1 = mr^2$, $\Theta_2 = \frac{1}{2}mr^2$

Kinematik: $\dot{x} = r\dot{\varphi}$, $\ddot{x} = r\ddot{\varphi}$
Reibgesetz: $R = \mu K$
Hohlzylinder ① :

$$
\begin{aligned}
m\ddot{x} &= K - H_1 + mg\sin\alpha && (1)\\
\Theta_1\ddot{\varphi} &= H_1 r - Rr && (2)
\end{aligned}
$$

Vollzylinder ② :

$$
\begin{aligned}
m\ddot{x} &= -K - H_2 + mg\sin\alpha && (3)\\
\Theta_2\ddot{\varphi} &= H_2 r - Rr && (4)
\end{aligned}
$$

$$\Longrightarrow \quad \boxed{\ddot{x} = \frac{4g\sin\alpha}{7+\mu}}$$

b)

$$
\begin{aligned}
\ddot{x} &= \frac{d\dot{x}}{dx}\dot{x}\\
\dot{x}d\dot{x} &= \frac{4g\sin\alpha}{7+\mu}dx
\end{aligned}
$$

$$\boxed{\dot{x}(x) = \sqrt{\frac{8g\sin\alpha}{7+\mu}x}}$$

c)

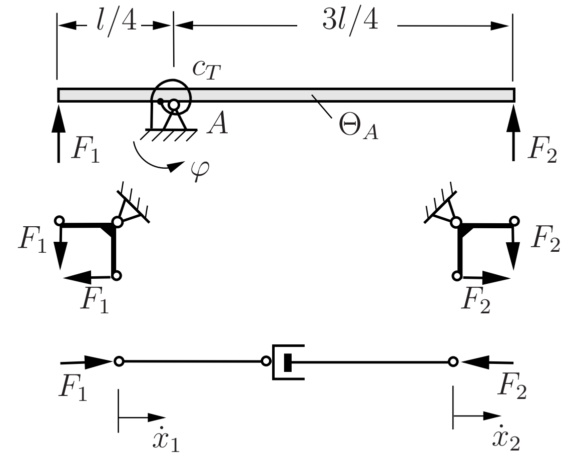
$$E_{\mathrm{P}_{(0)}} + E_{\mathrm{K}_{(0)}} = E_{\mathrm{P}_{(1)}} + E_{\mathrm{K}_{(1)}}$$

$$\begin{aligned}
E_{\mathrm{P}_{(0)}} &= 2mgx\sin\alpha \\
E_{\mathrm{K}_{(0)}} &= 0 \\
E_{\mathrm{P}_{(1)}} &= 0 \\
E_{\mathrm{K}_{(1)}} &= 2\,\frac{1}{2}m\dot{x}^2 + \frac{1}{2}\Theta_1\dot{\varphi}^2 + \frac{1}{2}\Theta_2\dot{\varphi}^2 \\
&= (1 + \frac{1}{2} + \frac{1}{2}\frac{1}{2})m\dot{x}^2 = \frac{7}{4}m\dot{x}^2
\end{aligned}$$

$$\begin{aligned}
\frac{7}{4}m\dot{x}^2 &= 2mgx\sin\alpha \\
\dot{x}(x) &= \sqrt{\frac{8g\sin\alpha}{7}x} \qquad \text{entspricht Ergebnis aus b) für } \mu = 0
\end{aligned}$$

Lösung - Aufgabe 2

a)
Freikörperbild:



Kräftegleichgewicht: und Dämpfergesetz

$$F_1 = F_2 = (\dot{x}_1 - \dot{x}_2)\, d \tag{1}$$

Trägheitsmoment

$$\Theta_A = \Theta_S + \left(\frac{l}{4}\right)^2 m = \left(\frac{1}{12} + \frac{1}{16}\right) ml^2 = \frac{7}{48} ml^2$$

Momentensatz:

$$\Theta_A \ddot{\varphi} = -\varphi\, c_t - F_1\, \frac{l}{4} + F_2\, \frac{3l}{4} \tag{2}$$

Geometrie (Kinematik):

$$\frac{l}{4}\, \varphi = x_1\,; \qquad \frac{l}{4}\, \dot{\varphi} = \dot{x}_1\,; \qquad \frac{l}{4}\, \ddot{\varphi} = \ddot{x}_1 \tag{3}$$

$$\frac{3l}{4}\, \varphi = x_2\,; \qquad \frac{3l}{4}\, \dot{\varphi} = \dot{x}_2\,; \qquad \frac{3l}{4}\, \ddot{\varphi} = \ddot{x}_2 \tag{4}$$

Einsetzen: (1), (3) und (4) in (2)

$$\Theta_A \ddot{\varphi} + \left(\frac{l}{2}\right)^2 d\dot{\varphi} + c_T \varphi = 0 \tag{5}$$

$$\boxed{\Theta_A \ddot{\varphi} + d\,\frac{l^2}{4}\dot{\varphi} + c_T\,\varphi = 0} \tag{6}$$

b)

Vergleich mit $\boxed{\ddot{x} + 2\delta\dot{x} + \omega^2 x = 0}$

liefert mit $\ddot{x} = \ddot{\varphi}$ $\quad \dot{x} = \dot{\varphi}$ $\quad x = \varphi$

$$\omega = \sqrt{\frac{c_T}{\Theta_A}}$$

$$\delta = D\omega = d\,\frac{l^2}{8\Theta_A}$$

$$\omega_d = \sqrt{\omega^2 - \delta^2}$$

c)

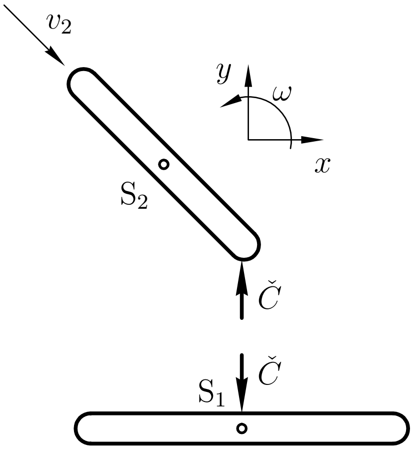
$$D = \frac{\delta}{\omega} = 1 \qquad \text{für aperiod. Grenzfall}$$

$$\Rightarrow \qquad \boxed{\delta = \omega}$$

$$\Rightarrow \qquad d = \frac{8}{l^2}\sqrt{c_T\Theta_A}$$

Lösung - Aufgabe 3

Freikörperbild



a) **Gleichungen**

Trägheitsmomente der Stangen bezüglich deren Schwerpunkte:

Stange 1: $\Theta_1^S = \dfrac{1}{12} m_1 (2a)^2 = \dfrac{1}{3} m_1 a^2$

Stange 2: $\Theta_2^S = \dfrac{1}{12} m_1 (2b)^2 = \dfrac{1}{3} m_2 b^2$

Drall- und Impulssatz:

$$\rightarrow: \quad m_1 (\bar{v}_{1Sx} - 0) = 0 \qquad (1)$$

$$\uparrow: \quad m_1 (\bar{v}_{1Sy} - 0) = -\check{C} \qquad (2)$$

$$\Theta_1^S (\bar{\omega}_1 - 0) = 0 \qquad (3)$$

$$\rightarrow: \quad m_2 (\bar{v}_{2Sx} - v_{2Sx}) = 0 \qquad (4)$$

$$\uparrow: \quad m_2 (\bar{v}_{2Sy} - v_{2Sy}) = \check{C} \qquad (5)$$

$$\Theta_2^S (\bar{\omega}_2 - 0) = \check{C} \frac{b}{\sqrt{2}} \qquad (6)$$

Kinematik:

$$v_{2Sx} = \frac{v_2}{\sqrt{2}} \qquad v_{2Sy} = -\frac{v_2}{\sqrt{2}}$$

$$v_{2Cx} = v_{2Sx} + \frac{b\,\omega_2}{\sqrt{2}} \qquad v_{2Cy} = v_{2Sy} + \frac{b\,\omega_2}{\sqrt{2}}$$

Stoßzahl:

$$e = -\frac{\bar{v}_{1Sy} - \bar{v}_{2Cy}}{v_{1Sy} - v_{2Cy}} \qquad (7)$$

b) **Berechnen von $\bar{\omega}_2$**

(7) mit Kinematik	$\bar{v}_{2Sy}$	$-$	$\bar{v}_{1Sy}$	$+$	$\frac{b}{\sqrt{2}}\,\bar{\omega}_2$	$=$	$-e\,v_{2Sy}$	(8)
$\frac{(3)}{m_1}+\frac{(1)}{m_1}$	$\frac{m_2}{m_1}\bar{v}_{2Sy}$	$+$	$\bar{v}_{1Sy}$			$=$	$\frac{m_2}{m_1}v_{2Sy}$	(9)
$\frac{1+\frac{m_2}{m_1}}{m_2}(3)-\frac{1+\frac{m_2}{m_1}}{m_2\,\frac{b}{\sqrt{2}}}(4)$	$\left(1+\frac{m_2}{m_1}\right)\bar{v}_{2Sy}$			$-$	$\frac{2}{3}\left(1+\frac{m_2}{m1}\right)\,\frac{b}{\sqrt{2}}\,\bar{\omega}_2$	$=$	$\left(1+\frac{m_2}{m1}\right)v_{2Sy}$	(10)
$(8)+(9)-(10)$					$\frac{1}{3}\left(5+\frac{2m_2}{m_1}\right)\frac{b}{\sqrt{2}}\,\bar{\omega}_2$	$=$	$-\,(e+1)\,v_{2Sy}$	

Ergebnis

$$\bar{\omega}_2=\frac{3\,v_2}{b}\,\frac{(e+1)\,m_1}{5\,m_1+2\,m_2}$$

Lösung - Aufgabe 4 / MB

a)

$$v_P = -v_F + l\omega_1 \tag{1}$$

$$r_1\omega_1 = r_2\omega_2 \tag{2}$$

$$v_F = R\omega_2 \tag{3}$$

$$\Longrightarrow \qquad v_P = -R\omega_2 + l\frac{r_2}{r_1}\omega_2 \tag{4}$$

$$\Longrightarrow \qquad \omega_2 = \frac{v_P}{\left(\frac{r_2}{r_1}\,l - R\right)} \tag{5}$$

$$\omega_1 = \frac{r_2}{r_1}\frac{v_P}{\left(\frac{r_2}{r_1}\,l - R\right)} \tag{6}$$

$$v_F = \frac{R\,v_P}{\frac{r_2}{r_1}\,l - R} \tag{7}$$

b)

Fahrrad nach links, d.h. $v_F > 0$:

$$\Longrightarrow \qquad \frac{l}{R}\,\frac{r_2}{r_1} > 1 \tag{8}$$

c)

Geschwindigkeit Punkt Q:

$$\vec{v}_Q = -v_F \vec{e}_x - \omega_2 R(\cos\varphi \vec{e}_x + \sin\varphi \vec{e}_y) \tag{9}$$

$$\vec{a}_Q = -\omega_2 R\dot{\varphi}(-\sin\varphi \vec{e}_x + \cos\varphi \vec{e}_y) \tag{10}$$

$$\dot{\varphi} = \omega_2 \tag{11}$$

$$\Longrightarrow \qquad \vec{a}_Q = R\omega_2^2(\sin\varphi \vec{e}_x - \cos\varphi \vec{e}_y)$$

Betrag: $\quad |\vec{a}_Q| = R\omega_2^2$

Richtung: $\quad \vec{a}_Q/|\vec{a}_Q| = \sin\varphi \vec{e}_x - \cos\varphi \vec{e}_y$

Lösung - Aufgabe 4 / BI

a) Satz von BERNOULLI: Umgebung (OK Wasser) $\rightarrow$ Düse

$$\rho g h = \frac{1}{2}\rho v_S^2 \qquad \rightsquigarrow \qquad v_S = \sqrt{2gh}$$

b) Impulssatz:

$$F = \dot{m} v_S\,, \qquad \dot{m} = \rho A_S v_S\,, \qquad A_S = \frac{\pi}{4}\, d_S^2$$

$$\rightsquigarrow \qquad F = \rho \frac{\pi}{4}\, d_S^2 v_S^2 = \rho \frac{\pi}{2}\, d_S^2\, gh$$

c) Trägheit / Momentenbilanz:

$$\Theta_W \ddot{\varphi}_0 = FR$$

$$\rightsquigarrow \qquad h = \frac{2\Theta_W \ddot{\varphi}_0}{\rho \pi d_S^2\, gR}$$