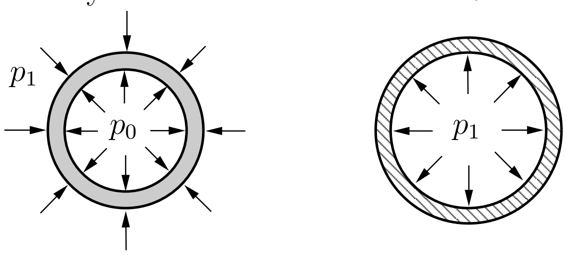
TM II Lösungen

Aufgabe 1

zu a)



zu b) Kesselformeln

$$\sigma_\varphi^{(1)} = (p_0 - p_1)\frac{r}{t}\ , \quad \sigma_x^{(1)} = \frac{p_0}{2}\ \frac{r}{t}\ , \quad \sigma_\varphi^{(2)} = p_1\frac{r}{t}\ , \quad \sigma_x^{(2)} = 0$$

zu c)

$$\varepsilon_x^{(1)} = \frac{1}{E_1}\left(\sigma_x^{(1)} - \nu_1\ \sigma_\varphi^{(1)}\right) = \frac{p_0 r}{2E_1 t}\left[1 - 2\nu_1\left(1 - \frac{p_1}{p_0}\right)\right]$$

$$\varepsilon_\varphi^{(1)} = \frac{1}{E_1}\left(\sigma_\varphi^{(1)} - \nu_1\ \sigma_x^{(1)}\right) = \frac{p_0 r}{2E_1 t}\left[2\left(1 - \frac{p_1}{p_0}\right) - \nu_1\right]$$

$$\varepsilon_x^{(2)} = \frac{1}{E_2}\left(\sigma_x^{(2)} - \nu_2\ \sigma_\varphi^{(2)}\right) = -\ \nu_2\frac{p_1 r}{E_2 t}$$

$$\varepsilon_\varphi^{(2)} = \frac{1}{E_2}\left(\sigma_\varphi^{(2)} - \nu_2\ \sigma_x^{(2)}\right) = \frac{p_1 r}{E_2 t}$$

zu d) Bedingung $\varepsilon_\varphi^{(1)} = \varepsilon_\varphi^{(2)}$

$$\rightsquigarrow\ \frac{p_1}{E_2} = \frac{p_0}{2E_1}\left[2\left(1 - \frac{p_1}{p_0}\right) - \nu_1\right] \quad \rightsquigarrow\ \underline{\underline{p_1 = p_0\frac{1 - \nu_1/2}{1 + E_1/E_2}}}$$

zu e)

$$\underline{\underline{\varepsilon_x^{(2)} = -\ p_0\frac{r}{t}\ \frac{\nu_2\,(1 - \nu_1/2)}{E_1 + E_2}}}$$

Aufgabe 2

zu a) Integration der Differentialgleichung der Biegelinie

$$\begin{aligned}
& EI\, w^{IV} &=& \; q_0 \sin(\tfrac{\pi}{2l}x) \\
-Q = \; & EI\, w''' &=& -q_0 \frac{2l}{\pi}\cos(\tfrac{\pi}{2l}x) + C_1 \\
-M = \; & EI\, w'' &=& -q_0 \left(\frac{2l}{\pi}\right)^2 \sin(\tfrac{\pi}{2l}x) + C_1 x + C_2 \\
& EI\, w' &=& \; q_0 \left(\frac{2l}{\pi}\right)^3 \cos(\tfrac{\pi}{2l}x) + C_1\frac{x^2}{2} + C_2 x + C_3 \\
& EI\, w &=& \; q_0 \left(\frac{2l}{\pi}\right)^4 \sin(\tfrac{\pi}{2l}x) + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4
\end{aligned}$$

Randbedingungen

$$\begin{aligned}
w(0) = 0 \; &: \quad C_4 = 0 \\
w''(0) = 0 \; &: \quad C_2 = 0 \\
w'(l) = 0 \; &: \quad C_1\frac{l^2}{2} + C_3 = 0 \\
c\, w(l) = -Q(l) \; &: \quad \frac{c}{EI}\left[q_0\left(\frac{2l}{\pi}\right)^4 + C_1\frac{l^3}{6} + C_3 l\right] = C_1
\end{aligned}$$

$$\rightsquigarrow \quad C_1 = \frac{q_0\left(\frac{2l}{\pi}\right)^4}{\frac{l^3}{3} + \frac{EI}{c}}\,, \qquad C_3 = -\frac{l^2}{2}\,\frac{q_0\left(\frac{2l}{\pi}\right)^4}{\frac{l^3}{3} + \frac{EI}{c}}$$

$$\underline{\underline{EI\, w(x) = q_0\left(\frac{2l}{\pi}\right)^4\left[\sin\frac{\pi}{2l}x + \frac{1}{2\left(1+\frac{3EI}{cl^3}\right)}\left(\frac{x}{l}\right)^3 - \frac{3}{2\left(1+\frac{3EI}{cl^3}\right)}\,\frac{x}{l}\right]}}$$

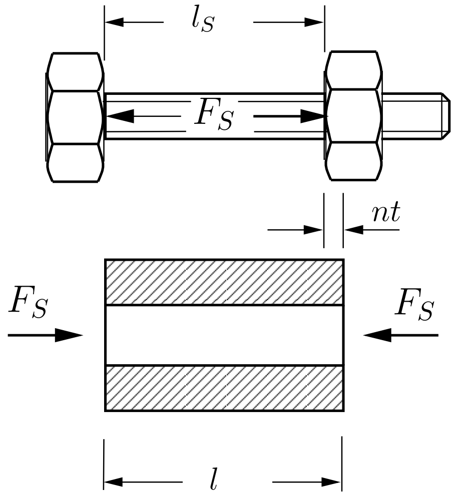
zu b)

$$B = Q(l) \qquad \text{oder} \qquad B = -c\, w(l)$$

$$\rightsquigarrow \quad \underline{\underline{B = -C_1 = -\frac{q_0\left(\frac{2l}{\pi}\right)^4}{\frac{l^3}{3} + \frac{EI}{c}}}}$$

Aufgabe 3

Freischneiden nach Anziehen der Mutter



Längenänderungen: $\Delta l_S = \frac{F_S l_S}{(EA)_S}$, $\Delta l_H = -\frac{F_S l}{(EA)_H}$

Kinematik $l_S + \Delta l_S = l + \Delta l_H$ mit $l_S = l - nt$

$$\rightsquigarrow \quad \Delta l_S = \Delta l_H + nt$$

zu a) Einsetzen von Δl_S und Δl_H liefert

$$\underline{\underline{F_S = \frac{nt}{\frac{l-nt}{(EA)_S} + \frac{l}{(EA)_H}}}}$$

zu b)

$$\underline{\underline{\Delta l_S = \frac{nt}{1 + \frac{l}{l-nt}\frac{(EA)_S}{(EA)_H}}}}$$

zu c)

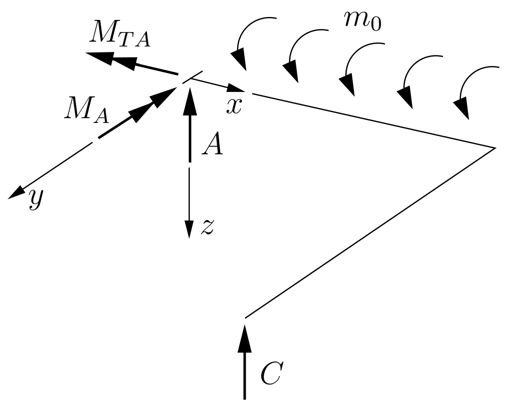
$$(EA)_H \to \infty \ : \quad \underline{\underline{F_S = \frac{nt\,(EA)_S}{l-nt}}}\ , \quad \underline{\underline{\Delta l_S = nt}}$$

$$(EA)_S \to \infty \ : \quad \underline{\underline{F_S = \frac{nt\,(EA)_H}{l}}}\ , \quad \underline{\underline{\Delta l_S = 0}}$$

Aufgabe 4

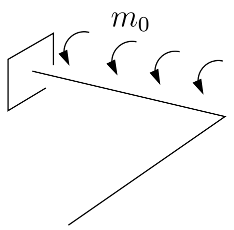
zu a) Lagerkraft C

Freischneiden:



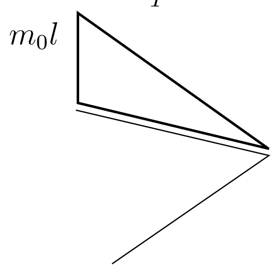
System ist 1-fach stat. unbestimmt, Bestimmung von C mit Prinzip der virtuellen Kräfte:

"0"-System

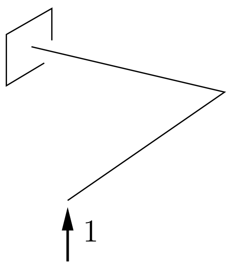


$$M^{(0)} = 0$$

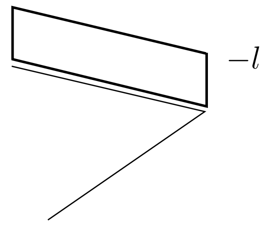
$M_T^{(0)}$-Linie



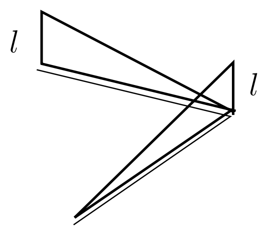
"1"-System



$M_T^{(1)}$-Linie



$M^{(1)}$-Linie



$$C = -\frac{\delta_{01}}{\delta_{11}}$$

$$\delta_{01} = \frac{1}{GI_T}\int M_T^{(0)} M_T^{(1)} \mathrm{d}x = -\frac{m_0 l^3}{2\, GI_T}$$

$$\delta_{11} = \frac{1}{GI_T}\int M_T^{(1)} M_T^{(1)} \mathrm{d}x + \frac{1}{EI}\int M^{(1)} M^{(1)} \mathrm{d}x$$

$$= \frac{l^3}{GI_T} + 2\frac{l^3}{3EI} = \frac{l^3}{GI_T}\left(1 + \frac{1}{3}\right) = \frac{4}{3}\,\frac{l^3}{GI_T}$$

$$\rightsquigarrow \quad \underline{\underline{C = \frac{3}{8}\, m_0}}$$

zu b) Torsionsverdrehung

$$GI_T\,\vartheta' = M_T \qquad\qquad M_T(x) = m_0(l-x) - Cl$$

$$= \frac{5}{8} m_0 l - m_0 x \qquad\qquad = \frac{5}{8} m_0 l - m_0 x$$

$$GI_T\,\vartheta = \frac{5}{8} m_0 l x - m_0 \frac{x^2}{2} + C_1$$

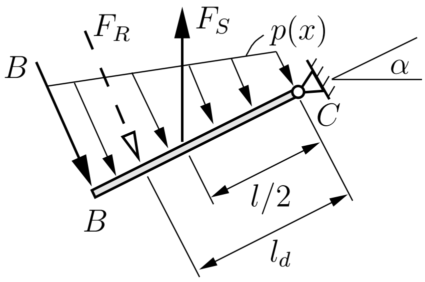
Randbedingungen: $\vartheta(0) = 0 \quad \rightsquigarrow \quad C_1 = 0$

$$\underline{\underline{\vartheta_B}} = \vartheta(l) = \frac{m_0 l^2}{GI_T}\left(\frac{5}{8} - \frac{1}{2}\right) = \underline{\underline{\frac{m_0 l^2}{8\, GI_T}}}$$

Aufgabe 4 (WI-BI)

zu a)

Schnittbild Klappe:



Gleichgewicht:

$$\sum M_C: \qquad B\,l - F_S\,\frac{l}{2}\cos\alpha + \frac{A}{l}\int\limits_0^l p(x)x\,\mathrm{d}x = 0$$

$$\text{mit} \qquad p(x) = \rho g(h + x\sin\alpha) \quad , \qquad \int\limits_0^l p(x)x\,\mathrm{d}x = \rho g l^2\left(\frac{h}{2} + \frac{l}{3}\sin\alpha\right)$$

oder :

$$\sum M_C: \qquad B\,l - F_S\,\frac{l}{2}\cos\alpha + F_R\,l_d = 0$$

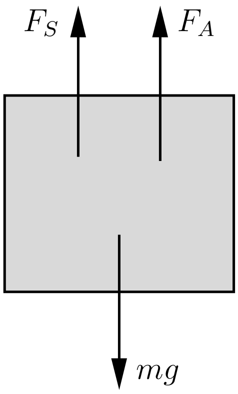
$$\text{mit} \qquad F_R = A\rho g h_s \quad , \qquad h_s = h + \frac{l}{2}\sin\alpha \quad , \qquad l_d = \frac{I}{S} - \frac{h}{\sin\alpha}$$

$$\text{und} \qquad I = \frac{Al^2}{12} + A\left(\frac{l}{2} + \frac{h}{\sin\alpha}\right)^2 \quad , \qquad S = A\left(\frac{l}{2} + \frac{h}{\sin\alpha}\right)$$

(Punkte nur auf einen der beiden Wege !)

$$\Longrightarrow \qquad B = \frac{1}{2}\,F_S\cos\alpha - A\rho g\left(\frac{h}{2} + \frac{l}{3}\sin\alpha\right)$$

Schnittbild Zylinder:



Gleichgewicht:

$$\uparrow: \quad F_s - mg + F_A = 0 \qquad \text{mit} \qquad F_A = A\rho g(h - h_0)$$

$$\Longrightarrow \quad F_S = mg - A\rho g(h - h_0)$$

$$\Longrightarrow \quad \underline{\underline{B = \frac{mg}{2}\cos\alpha - A\rho g\left(\frac{l}{3}\sin\alpha - \frac{h_0}{2}\cos\alpha + \frac{h}{2}(1+\cos\alpha)\right)}}$$

zu b)

Klappe geschlossen für $B > 0$. Klappe öffnet sich bei $B = 0$

$$\Longrightarrow \quad \underline{\underline{h = \frac{1}{1+\cos\alpha}\left(\frac{m\cos\alpha}{A\rho} + h_0\cos\alpha - \frac{2}{3}\,l\sin\alpha\right)}}$$