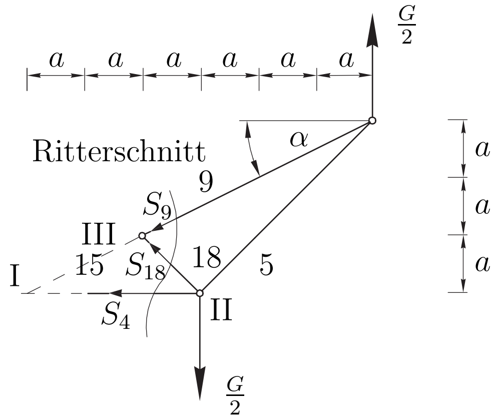
Lösung Aufgabe 1

1.) Nullstäbe: 12, 13, 14, 16, 17, 19, 20, 21

2.) Wegen der Nullstäbe und wegen der Symmetrie des Systems gilt:

$S_1 = S_2 = S_3 = S_5 = S_6 = S_7; \quad S_8 = S_9 = S_{10} = S_{11}; \quad S_{15} = S_{18}$



$\sum M_I = 0 = \frac{S_{18}}{\sqrt{2}} 3a - \frac{G}{2} 3a + \frac{G}{2} 6a$

$$\boxed{S_{15} = S_{18} = -\frac{G}{2}\sqrt{2}}$$

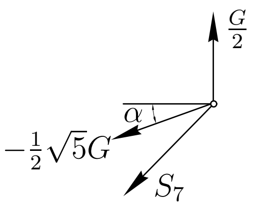
$\sum M_{II} = 0 = S_9 \sin\alpha \cdot 3a + \frac{G}{2} \cdot 3a$
mit $\arctan\alpha = \frac{1}{2}$

$$\boxed{S_8 = S_9 = S_{10} = S_{11} = -\frac{1}{2}\sqrt{5}G}$$

$\sum M_{III} = S_4 a + \frac{G}{2} a - \frac{G}{2} 4a = 0$

$$\boxed{S_4 = \frac{3}{2}G}$$

Freischnitt rechtes Lager:

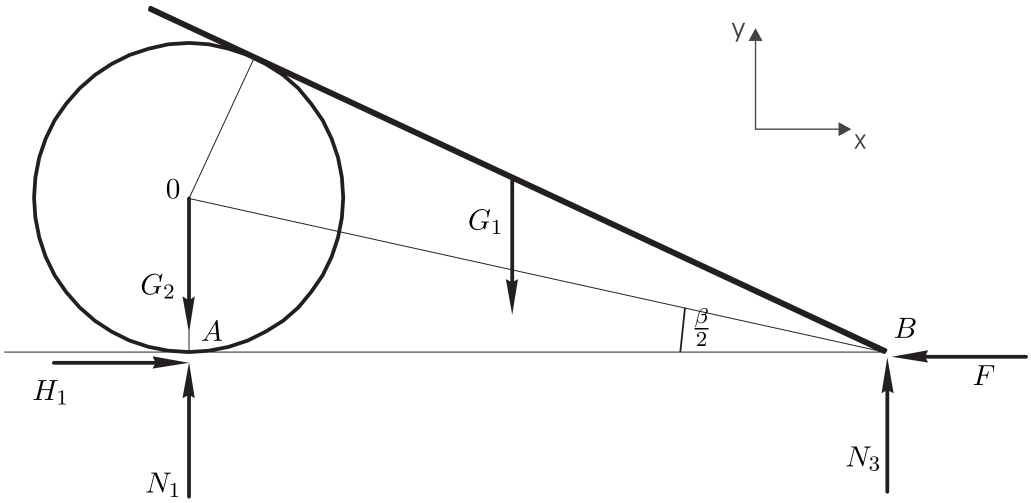


$\sum F_{iy} = \frac{1}{2}\sqrt{2} S_7 - \frac{1}{2}\sqrt{5}G \cdot \frac{1}{5}\sqrt{5} - \frac{G}{2} = 0$

$$\boxed{S_1 = S_2 = S_3 = S_5 = S_6 = S_7 = \sqrt{2}G}$$

Lösung zu Aufgabe 2:

1. Strecke $AB = R\cot\frac{\beta}{2} = \sqrt{3}R$



Gleichgewichtsbedingungen am Gesamtsystem:

$$\sum \quad F_{ix} = 0: \qquad H_1 - F = 0, \tag{1}$$

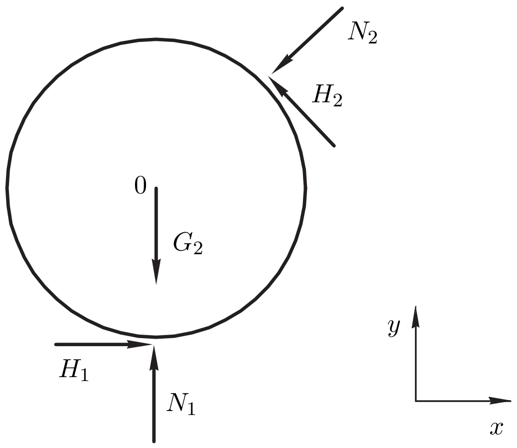
$$\sum \quad F_{iy} = 0: \qquad N_1 + N_3 - G_1 - G_2 = 0, \tag{2}$$

$$\sum \quad M^{(A)} = 0: \qquad G_1(R\cot\frac{\beta}{2} - \frac{L}{2}\cos\beta) - N_3 R\cot\frac{\beta}{2} = 0 \tag{3}$$

Also,

$$N_3 = \left(1 - \frac{L}{4\sqrt{3}R}\right)G \tag{4}$$

$$N_1 = G_1 + G_2 - N_3 = \left(1 + \frac{L}{4\sqrt{3}R}\right)G \tag{5}$$



Gleichgewichtsbedingungen an der Walze:

$$
\begin{aligned}
\sum F_{ix} = 0: &\qquad H_1 - H_2\cos\beta - N_2\sin\beta = 0, &(6)\\
\sum F_{iy} = 0: &\qquad -N_2\cos\beta - G_2 + H_2\sin\beta + N_1 = 0 &(7)\\
\sum M_i^{(O)} = 0: &\qquad RH_1 + RH_2 = 0 &(8)
\end{aligned}
$$

Auflösung der Gleichungen (6)-(8) unter Verwendung von (5) liefert

$$
\begin{aligned}
N_2 &= \frac{\sqrt{3}}{12}\frac{L}{R}G, &(9)\\
H_1 &= \frac{1}{12}\frac{L}{R}G, &(10)\\
H_2 &= -\frac{1}{12}\frac{L}{R}G &(11)
\end{aligned}
$$

Die Haftbedingungen sind

$$
\begin{aligned}
\mu_{01} &\geq \frac{|H_1|}{N_1} = \frac{\frac{1}{12}\frac{L}{R}}{1+\frac{\sqrt{3}}{12}\frac{L}{R}} = \frac{1}{\sqrt{3}+12\frac{R}{L}}, &(12)\\
\mu_{02} &\geq \frac{|H_2|}{N_2} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} &(13)
\end{aligned}
$$

2. Die Federkraft ist $F = H_1 = \frac{1}{12}\frac{L}{R}G$ und damit der Betrag der Längenänderung

$$
\Delta l_F = \left|-\frac{F}{c}\right| = \left|-\frac{1}{12}\frac{L}{cR}G\right| \qquad (14)
$$

3. Wegen

$$
\frac{1}{\sqrt{3}+12\frac{R}{L}} < \frac{1}{\sqrt{3}}
$$

tritt Gleiten an der Stelle 2 zunächst zwischen Balken und Walze auf, wenn $\mu_{01} = \mu_{02} = \mu_0$ ist.

Lösung zu Aufgabe 3

1)
$$
\begin{aligned}
x_2 &= l \sin\varphi \\
z_2 &= k\frac{x_2^2}{l} = kl\sin^2\varphi \\
z_1 &= z_2 + l\cos\varphi = kl\sin^2\varphi + l\cos\varphi \\
\Delta l_F &= \frac{1}{2} l \sin\varphi
\end{aligned}
$$

$$
\begin{aligned}
E_{pot} = \Pi &= Gz_1 + 2Gz_2 + \frac{1}{2}c(\Delta l_F)^2 \\
&= Gl\left(k\sin^2\varphi + \cos\varphi\right) + 2Gl\left(k\sin^2\varphi\right) + \frac{1}{2}8\frac{G}{l}\left(\frac{1}{2}l\sin\varphi\right)^2 \\
&= Gl\left[(3k+1)\sin^2\varphi + \cos\varphi\right]
\end{aligned}
$$

$$
\frac{\partial\Pi}{\partial\varphi} = Gl\left[(6k+2)\sin\varphi\cos\varphi - \sin\varphi\right] \overset{!}{=} 0
$$

$$
a) \qquad \sin\varphi_{1/2} \overset{!}{=} 0 \rightsquigarrow \begin{cases} \varphi_1 = 0^\circ \\ \varphi_2 = 180^\circ \end{cases}
$$

$$
b) \qquad (6k+2)\cos\varphi_3 - 1 \overset{!}{=} 0 \rightsquigarrow \cos\varphi_3 = \frac{1}{6k+2}
$$

2)
$$
\begin{aligned}
\frac{\partial^2\Pi}{\partial\varphi^2} &= Gl\left[(6k+2)(\cos^2\varphi - \sin^2\varphi) - \cos\varphi\right] \\
&= Gl\left[(6k+2)(2\cos^2\varphi - 1) - \cos\varphi\right]
\end{aligned}
$$

$$
\varphi_1 = 0^\circ \quad : \quad \left.\frac{\partial^2\Pi}{(\partial\varphi)^2}\right|_{\varphi=0^\circ} = (6k+2) - 1 \overset{!}{>} 0
$$

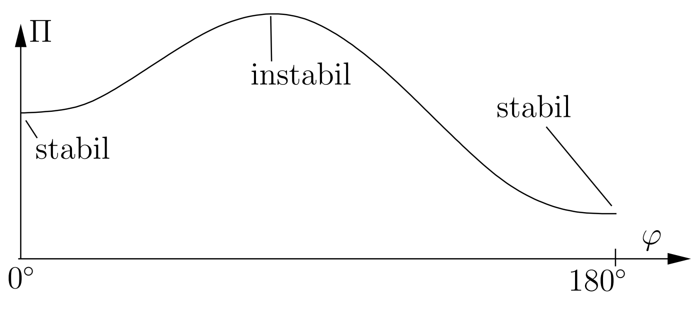
$$
\rightsquigarrow \text{ stabil für } k > -\frac{1}{6}
$$

$$
\varphi_1 = 180^\circ \quad : \quad \left.\frac{\partial^2\Pi}{(\partial\varphi)^2}\right|_{\varphi=180^\circ} = (6k+2) + 1 \overset{!}{>} 0
$$

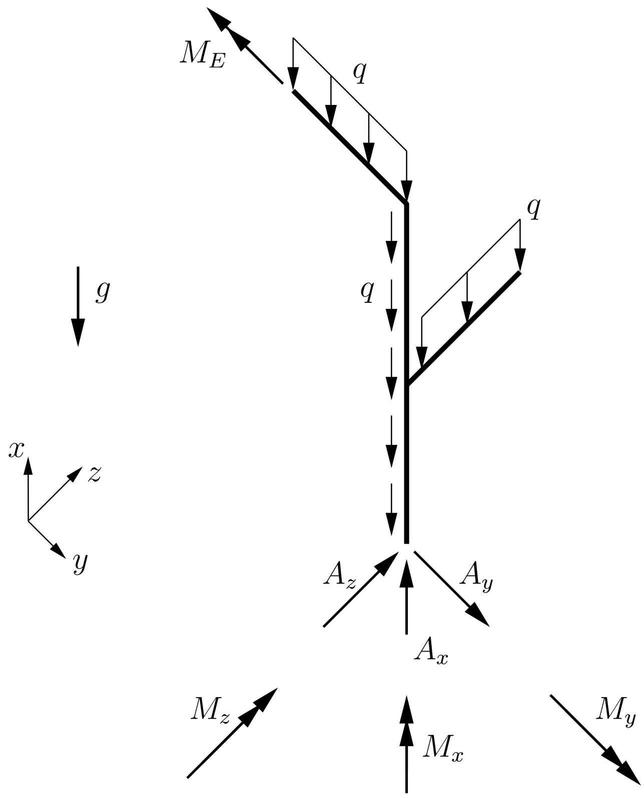
$$
\rightsquigarrow \text{ stabil für } k > -\frac{1}{2}
$$

also muss gelten: $k > -\frac{1}{6}$

3) Potentialverlauf für $k = 1$:



Lösung zu Aufgabe 4



Gleichgewichtsbedingungen:

$$\sum F_{ix} = 0: \qquad A_x - 4qa = 0 \qquad \rightarrow \qquad A_x = 4qa$$

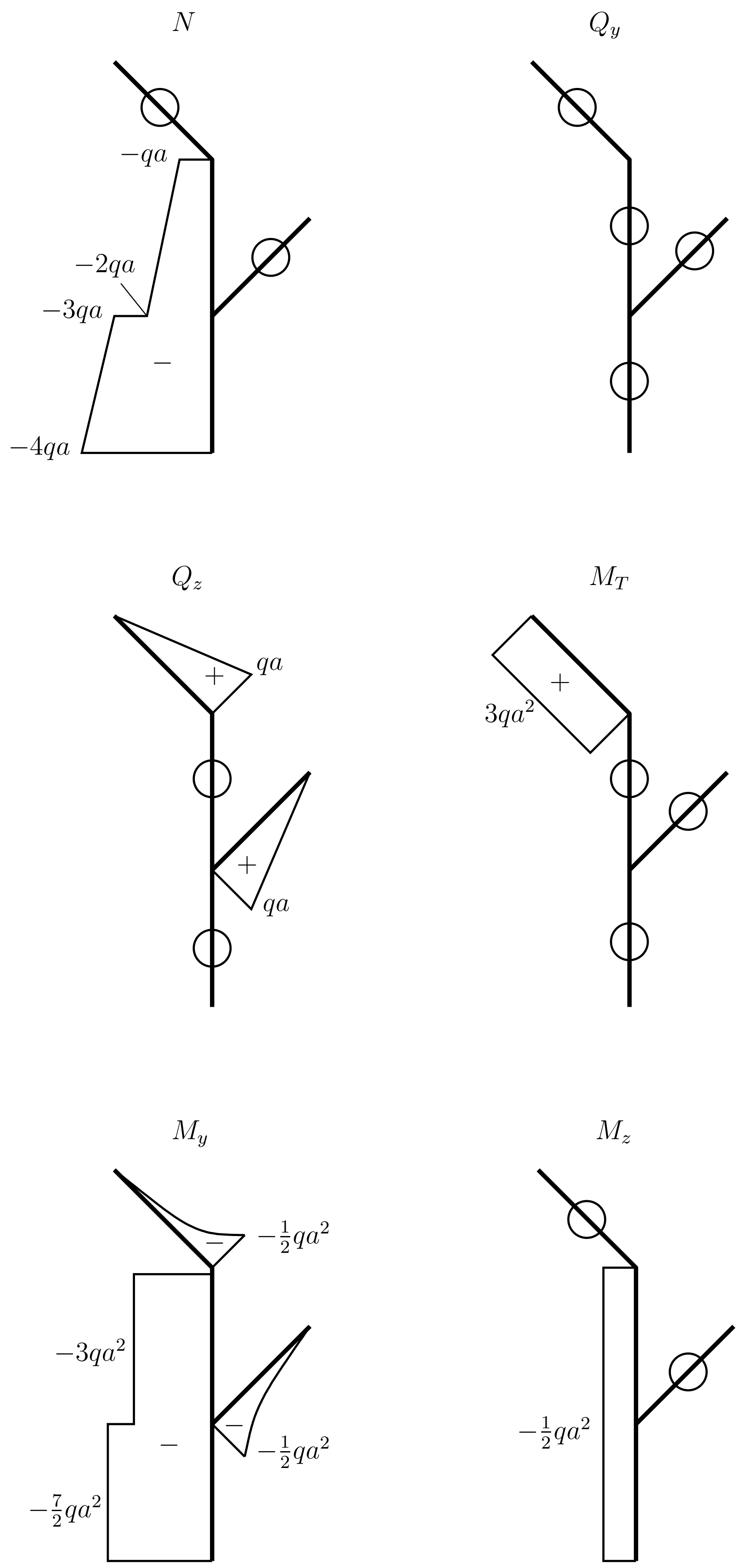
$$\sum F_{iy} = 0: \qquad A_y = 0$$

$$\sum F_{iz} = 0: \qquad A_z = 0$$

$$\sum M_{ix}^A = 0: \qquad M_x = 0$$

$$\sum M_{iy}^A = 0: \qquad M_y - \frac{1}{2}qa^2 - M_E = 0 \qquad \rightarrow \qquad M_y = \frac{7}{2}qa^2$$

$$\sum M_{iz}^A = 0: \qquad M_z - \frac{1}{2}qa^2 = 0 \qquad \rightarrow \qquad M_z = \frac{1}{2}qa^2$$


N
$-qa$
$-2qa$
$-3qa$
$-$
$-4qa$
Q_y
Q_z
qa
$+$
$+$
qa
M_T
$+$
$3qa^2$
M_y
$-$
$-\frac{1}{2}qa^2$
$-3qa^2$
$-$
$-$
$-\frac{1}{2}qa^2$
$-\frac{7}{2}qa^2$
M_z
$-\frac{1}{2}qa^2$