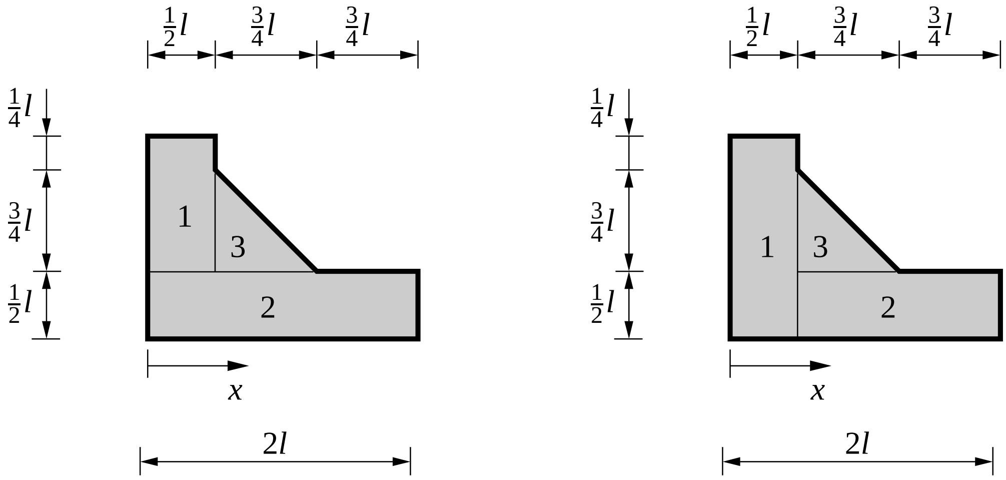
Aufgabe 1 [21 Punkte]

a) Schwerpunktskoordinate x_s (2 Alternativen):



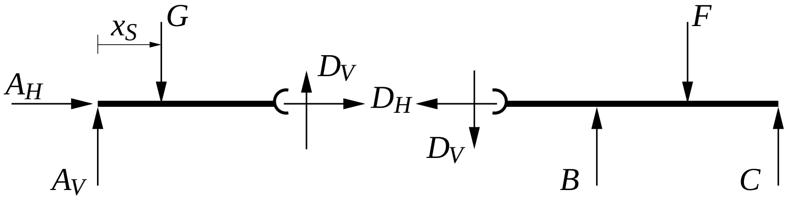
i	x_i	A_i	x_iA_i
1	$\frac{1}{4}l$	$l \cdot \frac{1}{2}l = \frac{1}{2}l^2$	$\frac{1}{8}l^3$
2	l	$2l \cdot \frac{1}{2}l = l^2$	l^3
3	$\frac{1}{2}l + \frac{1}{3}\frac{3}{4}l = \frac{3}{4}l$	$\frac{1}{2}\left(\frac{3}{4}l\right)^2 = \frac{9}{32}l^2$	$\frac{27}{128}l^3$
Σ		$\frac{57}{32}l^2$	$\frac{171}{128}l^3$

i	x_i	A_i	x_iA_i
1	$\frac{1}{4}l$	$\frac{3}{2}l \cdot \frac{1}{2}l = \frac{3}{4}l^2$	$\frac{3}{16}l^3$
2	$\frac{1}{2}l + \frac{3}{4}l = \frac{5}{4}l$	$\frac{3}{2}l \cdot \frac{1}{2}l = \frac{3}{4}l^2$	$\frac{15}{16}l^3$
3	$\frac{1}{2}l + \frac{1}{3}\frac{3}{4}l = \frac{3}{4}l$	$\frac{1}{2}\left(\frac{3}{4}l\right)^2 = \frac{9}{32}l^2$	$\frac{27}{128}l^3$
Σ		$\frac{57}{32}l^2$	$\frac{171}{128}l^3$

$$x_S = \frac{\sum x_i A_i}{\sum A_i} \qquad \Rightarrow \qquad \boxed{x_s = \frac{3}{4}l}$$

b) Auflager- und Gelenkkräfte:

<u>FKB:</u>



$$\rightarrow_{\text{gesamt}}: \quad \boxed{A_H = 0}$$

$$\rightarrow_{\text{links}}: \quad \boxed{D_H = 0}$$

$$\overset{\frown}{A}_{\text{links}}: \quad -Gx_S + D_V 2l = 0 \quad \Rightarrow \quad \boxed{D_V = \frac{x_S}{2l}G = \frac{3}{8}G}$$

$$\overset{\frown}{D}_{\text{links}}: \quad -A_V 2l + G(2l - x_S) = 0 \quad \Rightarrow \quad \boxed{A_V = \frac{2l - x_S}{2l}G = \left(1 - \frac{x_S}{2l}\right)G = \frac{5}{8}G}$$

$$\uparrow_{\text{links}}: \quad A_V + D_V - G = 0 \quad \Rightarrow \quad \boxed{D_V = G - A_V = \frac{x_S}{2l}G = \frac{3}{8}G}$$

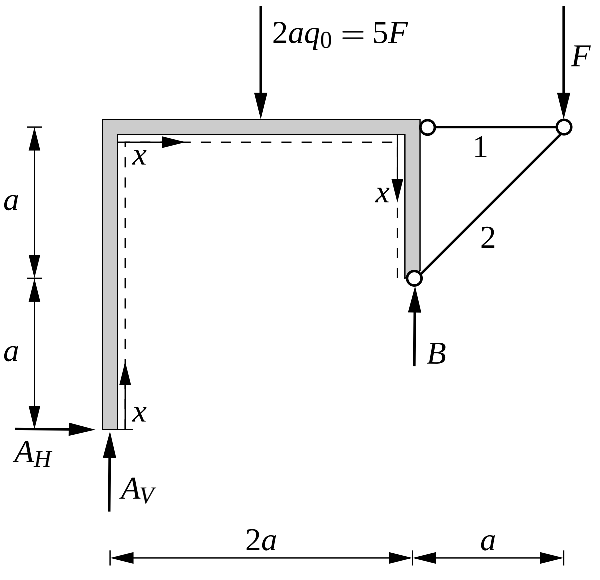
$$\Rightarrow \quad \boxed{A_V = G - D_V = \left(1 - \frac{x_S}{2l}\right)G = \frac{5}{8}G}$$

$$\overset{\frown}{C}_{\text{rechts}}: \quad D_V 3l - B2l + Fl = 0 \quad \Rightarrow \quad \boxed{B = \frac{1}{2}F + \frac{3}{2}D_V = \frac{1}{2}F + \frac{9}{16}G}$$

$$\uparrow_{\text{rechts}}: \quad B + C - D_V - F = 0 \quad \Rightarrow \quad \boxed{C = \frac{1}{2}F - \frac{1}{2}D_V = \frac{1}{2}F - \frac{3}{16}G}$$

Aufgabe 2 [30 Punkte]

a) Lagerreaktionen und Stabkräfte:



Gleichgewicht am Gesamtsystem:

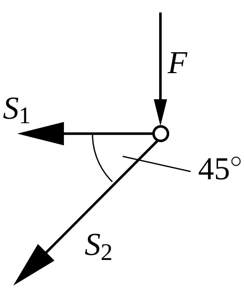
$$\rightarrow: \quad \boxed{A_H = 0}$$

$$\overset{\frown}{A}: \quad -5Fa + B2a - F3a = 0$$

$$\Rightarrow \quad \boxed{B = 4F}$$

$$\uparrow: \quad A_V + B - 5F - F = 0$$

$$\Rightarrow \quad \boxed{A_V = 2F}$$



Knotengleichgewicht:

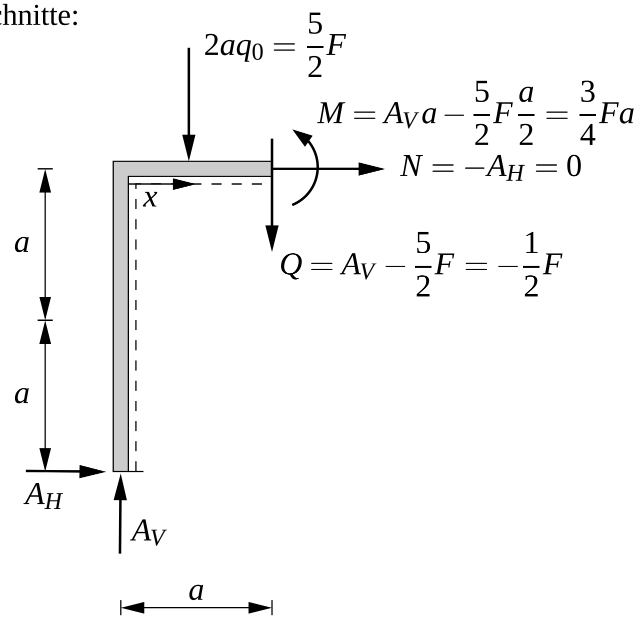
$$\uparrow: \quad -\frac{1}{\sqrt{2}}S_2 - F = 0$$

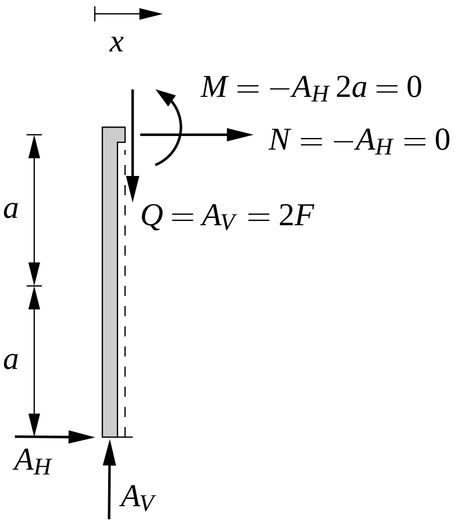
$$\Rightarrow \quad \boxed{S_2 = -\sqrt{2}F}$$

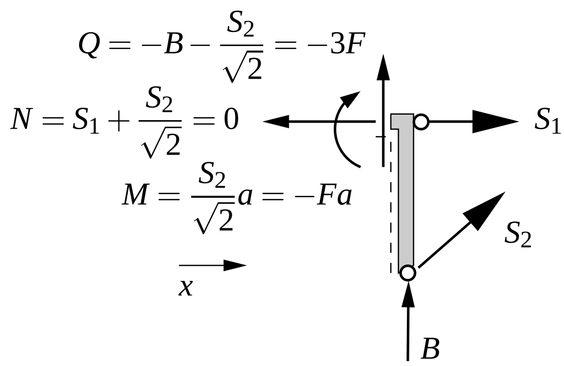
$$\rightarrow: \quad -S_1 - \frac{1}{\sqrt{2}}S_2 = 0$$

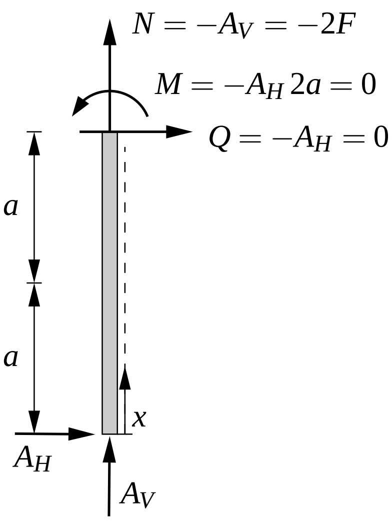
$$\Rightarrow \quad \boxed{S_1 = F}$$

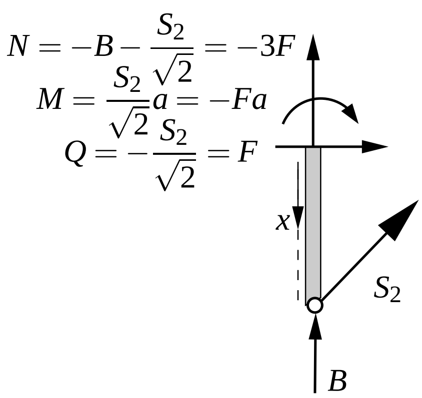
b) Schnitte:


$2aq_0 = \frac{5}{2}F$
$M = A_V a - \frac{5}{2}F\frac{a}{2} = \frac{3}{4}Fa$
$N = -A_H = 0$
$Q = A_V - \frac{5}{2}F = -\frac{1}{2}F$
x
a
a
A_H
A_V
a

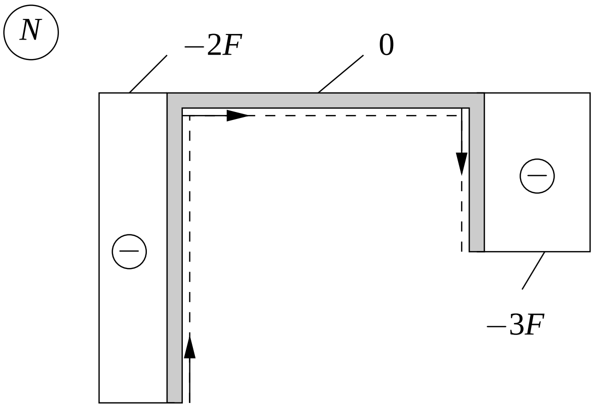

x
$M = -A_H\,2a = 0$
$N = -A_H = 0$
$Q = A_V = 2F$
a
a
A_H
A_V

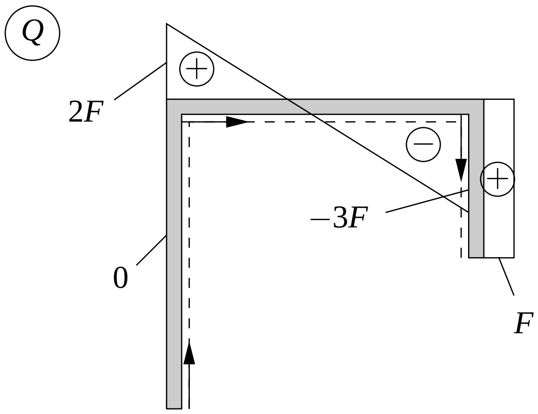

$Q = -B - \frac{S_2}{\sqrt{2}} = -3F$
$N = S_1 + \frac{S_2}{\sqrt{2}} = 0$
S_1
$M = \frac{S_2}{\sqrt{2}}a = -Fa$
S_2
x
B

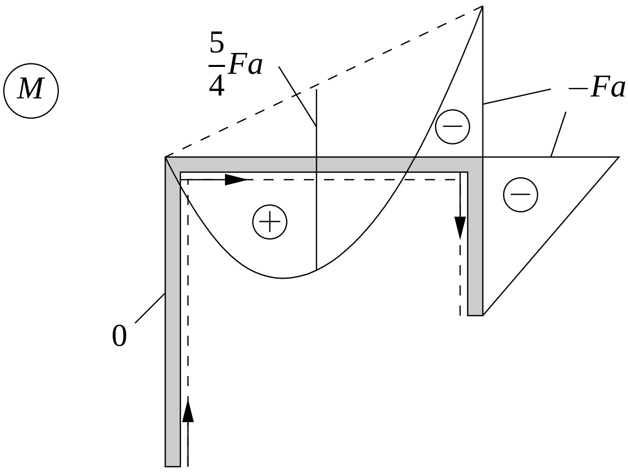

$N = -A_V = -2F$
$M = -A_H\,2a = 0$
$Q = -A_H = 0$
a
a
x
A_H
A_V


$N = -B - \frac{S_2}{\sqrt{2}} = -3F$
$M = \frac{S_2}{\sqrt{2}}a = -Fa$
$Q = -\frac{S_2}{\sqrt{2}} = F$
x
S_2
B

Schnittgrößen:

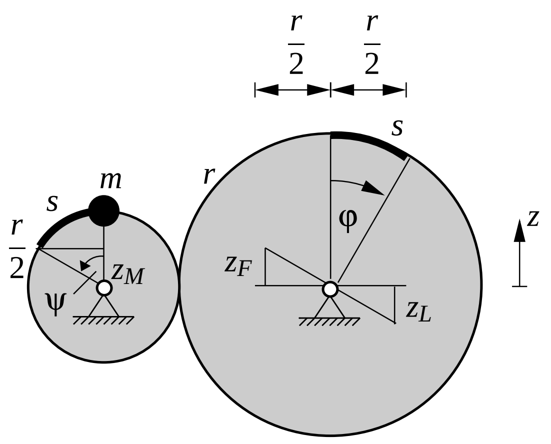






Aufgabe 3 [20 Punkte]

a) Potential:



Kinematik:

große Rolle: $\varphi = \frac{s}{r}$ (Bogenmaß)

kleine Rolle: $\psi = \frac{s}{\frac{r}{2}} = 2\frac{s}{r} = 2\varphi$

z-Koordinaten von Masse, Loch und Federangriffspunkt:

$$\begin{aligned} z_M &= \frac{r}{2}\cos\psi = \frac{r}{2}\cos(2\varphi) \\ z_L &= -\frac{r}{2}\sin\varphi \\ z_F = \Delta l_F &= \frac{r}{2}\sin\varphi \end{aligned}$$

$$\begin{aligned} \Pi &= -8mg(-\frac{r}{2}\sin\varphi) + \frac{1}{2}c\left(\frac{r}{2}\sin\varphi\right)^2 + mg\frac{r}{2}\cos(2\varphi) \\ &= 8mg\frac{r}{2}\sin\varphi + \frac{1}{2}c\left(\frac{r}{2}\sin\varphi\right)^2 + mg\frac{r}{2}\cos(2\varphi) \\ &= 4mgr\sin\varphi + \frac{1}{2}\cdot 40\frac{mg}{r}\frac{r^2}{4}\sin^2\varphi + mg\frac{r}{2}\cos(2\varphi) \\ &= mgr\left(4\sin\varphi + 5\sin^2\varphi + \frac{1}{2}\cos(2\varphi)\right) \end{aligned}$$

$$\begin{aligned} \frac{d\Pi}{d\varphi} &= mgr\,(4\cos\varphi + 10\sin\varphi\cos\varphi - \sin(2\varphi)) \\ &= 4mgr\cos\varphi\,(1 + 2\sin\varphi) \end{aligned}$$

Gleichgewicht: $\frac{d\Pi}{d\varphi} = 0$:

$\cos\varphi = 0$ oder $1 + 2\sin\varphi = 0$

$\Longrightarrow \varphi_1 = 90^\circ$, $\varphi_2 = 270^\circ$
$\Longrightarrow \varphi_3 = 210^\circ$, $\varphi_4 = 330^\circ$

b) Stabilität:

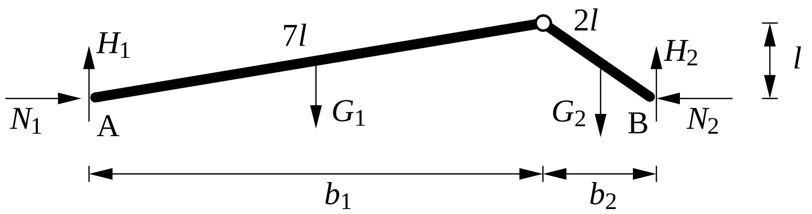
$$\begin{aligned}\frac{d^2\Pi}{d\varphi^2} &= -4mgr\sin\varphi\,(1+2\sin\varphi)+8mgr\cos^2\varphi \\ &= 4mgr\left(2\cos^2\varphi-\sin\varphi-2\sin^2\varphi\right)\end{aligned}$$

Gleichgewichtslagen einsetzten:

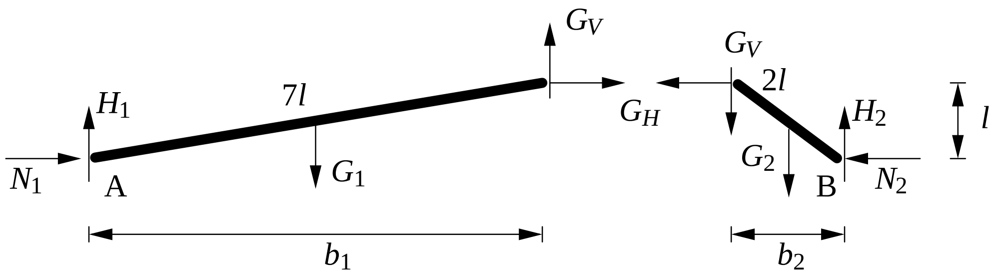
$$\begin{aligned}\left.\frac{d^2\Pi}{d\varphi^2}\right|_{\varphi=\varphi_1} &= 4mgr\,(0-1-2)<0 \quad \Longrightarrow \text{instabil} \\ \left.\frac{d^2\Pi}{d\varphi^2}\right|_{\varphi=\varphi_2} &= 4mgr\,(0-1\cdot(-1)-2)<0 \quad \Longrightarrow \text{instabil} \\ \left.\frac{d^2\Pi}{d\varphi^2}\right|_{\varphi=\varphi_3} &= 4mgr\left(\left(\frac{\sqrt{3}}{2}\right)^2\cdot 2+\frac{1}{2}-\frac{1}{2}\right)>0 \quad \Longrightarrow \text{stabil} \\ \left.\frac{d^2\Pi}{d\varphi^2}\right|_{\varphi=\varphi_4} &= \left.\frac{d^2\Pi}{d\varphi^2}\right|_{\varphi=\varphi_3}>0 \quad \Longrightarrow \text{stabil}\end{aligned}$$

Aufgabe 4 [19 Punkte]

Freikörperbild: (Gesamtsystem)



Freikörperbild: (Teilsysteme)



Geometrie:

$$b_1 = \sqrt{(7l)^2 - l^2} = \sqrt{48}l = 4\sqrt{3}l$$
$$b_2 = \sqrt{(2l)^2 - l^2} = \sqrt{3}l$$

- Alternative 1: Gleichgewicht am Gesamtsystem:

$$\overset{\frown}{A}: \quad H_2(b_1 + b_2) - G_1\frac{b_1}{2} - G_2\left(b_1 + \frac{b_2}{2}\right) = 0$$

$$\Rightarrow \quad 5\sqrt{3}l \cdot H_2 - 2\sqrt{3}l \cdot G - \frac{9}{2}\sqrt{3}l \cdot \frac{G}{3} = 0$$

$$\Leftrightarrow \quad \boxed{H_2 = \frac{7}{10}G}$$

$$\overset{\frown}{B}: \quad -H_1(b_1 + b_2) + G_1\left(b_2 + \frac{b_1}{2}\right) + G_2\frac{b_2}{2} = 0$$

$$\Rightarrow \quad -5\sqrt{3}l \cdot H_1 + 3\sqrt{3}l \cdot G + \frac{\sqrt{3}}{2}l \cdot \frac{G}{3} = 0$$

$$\Leftrightarrow \quad \boxed{H_1 = \frac{19}{30}G}$$

$$\rightarrow: \quad N_1 = N_2$$

Gleichgewicht am linken Teilsystem:

$$\overset{\frown}{G}: \quad -H_1 b_1 + N_1 l + G_1 \frac{b_1}{2} = 0$$

$$\Rightarrow \quad -4\sqrt{3}l \cdot H_1 + N_1 l + 2\sqrt{3}lG = 0$$

$$\Rightarrow \quad N_1 = 4\sqrt{3}\frac{19}{30}G - 2\sqrt{3}G = \frac{8}{15}\sqrt{3}G$$

$$\Rightarrow \quad \boxed{N_1 = N_2 = \frac{8}{15}\sqrt{3}G}$$

oder

Gleichgewicht am rechten Teilsystem:

$$\overset{\frown}{G}: \quad H_2 b_2 - N_2 l - G_2 \frac{b_2}{2} = 0$$

$$\Rightarrow \quad \sqrt{3}l \cdot H_2 - N_2 l - \frac{\sqrt{3}}{2}\frac{G}{3} = 0$$

$$\Rightarrow \quad N_2 = \sqrt{3}\frac{7}{10}G - \frac{\sqrt{3}}{6}G = \frac{8}{15}\sqrt{3}G$$

$$\Rightarrow \quad \boxed{N_1 = N_2 = \frac{8}{15}\sqrt{3}G}$$

- Alternative 2: nur Gleichgewicht an Teilsystemen:

$$\begin{aligned}
&\text{links} && \rightarrow: && N_1 + G_H = 0 && (1)\\
& && \uparrow: && H_1 + G_V - G = 0 && (2)\\
& && \overset{\frown}{A}: && -\frac{b_1}{2}G + b_1 G_V - lG_H = 0 && (3)\\
&\text{rechts} && \rightarrow: && -N_2 - G_H = 0 && (4)\\
& && \uparrow: && H_2 - G_V - \frac{1}{3}G = 0 && (5)\\
& && \overset{\frown}{B}: && -\frac{b_2}{2}\frac{G}{3} + b_2 G_V + lG_H = 0 && (6)
\end{aligned}$$

$$(3)+(6) \quad \Rightarrow \quad G_V = \frac{11}{30}G$$

$$(2) \quad \Rightarrow \quad H_1 = G - G_V \quad \Rightarrow \quad \boxed{H_1 = \frac{19}{30}G}$$

$$(5) \quad \Rightarrow \quad H_2 = \frac{1}{3}G + G_V \quad \Rightarrow \quad \boxed{H_2 = \frac{7}{10}G}$$

$$(3)\,\text{oder}\,(6) \quad \Rightarrow \quad G_H = -\frac{8}{15}\sqrt{3}G$$

$$(1)\,\text{und}\,(4) \quad \Rightarrow \quad N_1 = N_2 = -G_H \quad \Rightarrow \quad \boxed{N_1 = N_2 = \frac{8}{15}\sqrt{3}G}$$

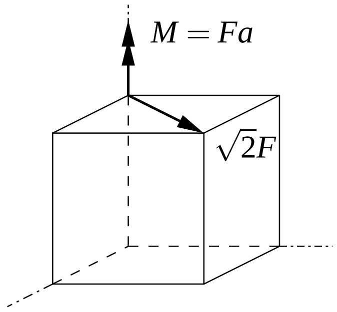
Haftbedingungen:

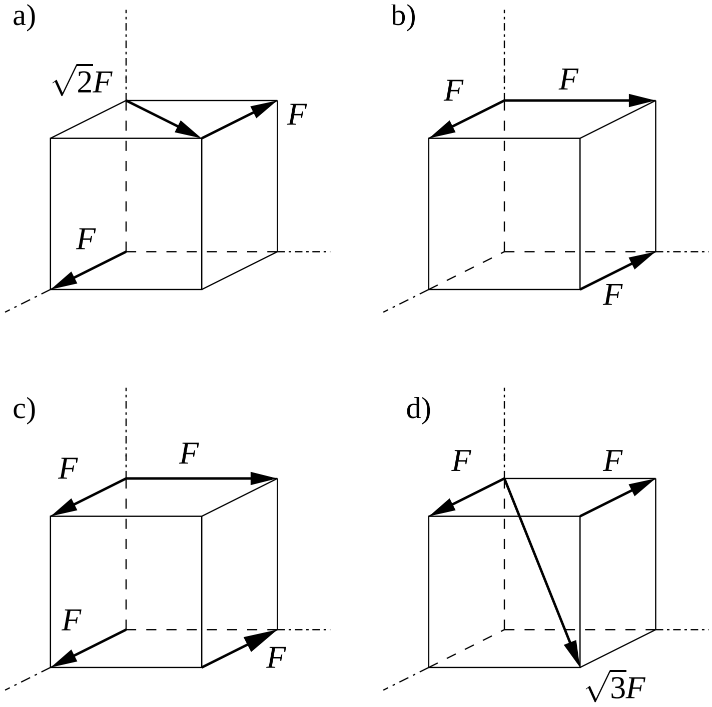
$$\text{Es muss gelten:} \quad H_1 \leq \mu_0 N_1 \quad \Rightarrow \quad \mu_0 \geq \frac{H_1}{N_1} = \frac{\frac{19}{30}G}{\frac{8\sqrt{3}}{15}G} = \frac{19\sqrt{3}}{48} = \mu^*$$

$$\text{und:} \quad H_2 \leq \mu_0 N_2 \quad \Rightarrow \quad \mu_0 \geq \frac{H_2}{N_2} = \frac{\frac{7}{10}G}{\frac{8\sqrt{3}}{15}G} = \frac{7\sqrt{3}}{16} > \mu^*$$

$$\Rightarrow \quad \boxed{\mu_0^{\min} = \frac{7\sqrt{3}}{16}}$$

Kurzfrage 1 [4 Punkte]





Antwort: c)

Kurzfrage 2 [2 Punkte]

$$F = \frac{G}{4}$$

Kurzfrage 3 [4 Punkte]